Active Sterile Neutrino Conversions in a Supernova with Random Magnetic Fields

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Abstract

Large enough random magnetic fields may affect in an important way neutrino conversion rates, even in the case where neutrinos have zero transition magnetic moments. We consider their effect in the case of active to sterile neutrino conversions in a supernova and show that for KeV neutrino masses these limits may overcome those derived for the case of zero magnetic field.

1. Introduction

There have been several hints for nonzero neutrino masses from astrophysical and comological observations which, taken altogether, point towards a class of extensions of the standard model that contain a light sterile neutrino [1].

So far the most stringent constraints for the neutrino mass matrix including a fourth neutrino species, ν_s, come from the nucleosynthesis bound on the maximum number of extra neutrino species that can reach thermal equilibrium before nucleosynthesis and change the primordially produced helium abundance [2]. This has been widely discussed in the case of the early Universe hot plasma without magnetic field, as well as recently for the case of a large random magnetic field (r.m.f.) [3].

Stringent constraints on the active to sterile neutrino oscillation parameters have been derived for the case of supernovae with zero magnetic field in ref. [4]. Here we summarize the results of ref. [5] on the effect that a large supernova r.m.f. has on the active sterile neutrino conversions. This was motivated by a recent paper [6] which showed that magnetic fields as strong as 10^{14} to 10^{16} Gauss might be generated during the first seconds of neutrino emission inside a supernova core. If such field is generated after collapse it could be viewed as the random superposition of many small dipoles with size $L_0 \sim 1$ Km [6]. Although the magnetic field in different domains is randomly aligned relative to the neutrino propagation direction, the neutrino conversion probabilities depend on the mean-squared random field via a squared magnetization value, leading therefore to nonvanishing averages over the magnetic field distribution.

The effect which we have found is of more general validity than that which could be ascribed to nonzero magnetic (transition) moments, as it would exist even these are negligible, as expected in the simplest extensions of the standard model.

2. Active-sterile neutrino conversions in the presence of a large r.m.f.

The equation of motion for a system of one active and one sterile neutrinos propagating in the presence of a large r.m.f. can be written in terms of weak eigenstates, as

$$i\frac{d}{dt}\begin{pmatrix}\nu_a\\ \nu_s\end{pmatrix} = \begin{pmatrix}H_{aa} & H_{as}\\ H_{as} & H_{ss}\end{pmatrix}\begin{pmatrix}\nu_a\\ \nu_s\end{pmatrix}, \qquad (2.1)$$

where the quantities in the evolution hamiltonian are

given as

$$H_{aa} = (c^2 m_1^2 + s^2 m_2^2)/2q + V_{as} + A_{as} \qquad (2.2)$$
$$H_{as} = cs\Delta$$
$$H_{ss} = (s^2 m_1^2 + c^2 m_2^2)/2q$$

and we have denoted by V_{as} and A_{as} the vector and axial parts of the neutrino potential that will describe the active to sterile conversions, given as

$$V_{as} \approx 4 \times 10^{-6} \rho_{14}(3Y_e + 4Y_{\nu_e} - 1)\mathrm{MeV}, \qquad (2.3)$$

$$A_{as}(q, B) = V_{axial}\frac{q_z}{q} \qquad (2.4)$$

where the term V_{axial} is produced by the *mean axial current* and is proportional to the magnetization of the plasma in the external magnetic field, assumed to be pointed along the z-direction inside a given domain. In the above equations q is the neutrino momentum, m_1 and m_2 are the masses of the neutrinos , θ is their mixing angle and we use the standard definitions $\Delta = \Delta m^2/2q$; $\Delta m^2 = m_2^2 - m_1^2$; $c = \cos\theta$, and $s = \sin\theta$. The Y's denote particle abundances and ρ_{14} denotes the density in units of 10^{14} g/cc.

Notice that, although majorana neutrinos could have nonzero transition magnetic moments [7], we have neglected them in our present discussion. As we will see, even in this case, there may be a large effect of the magnetic field on the conversion rates.

From (2.1) one can easily obtain the probability $P_{\nu_a \to \nu_s}(t)$ for converting the active neutrinos ν_a emitted by the supernova into the sterile neutrinos, ν_s. In a strong random magnetic field one can write

$$P_{\nu_a \to \nu_s}(B, t) \approx \frac{\Delta^2 \sin^2 2\theta}{2\Delta_m^2}\Big(1 - \exp(-\Delta_m^2 t/2\Gamma)\Big), \qquad (2.5)$$

which describes the aperiodic behaviour of the active to sterile neutrino conversion. The relaxation time defined as

$$t_{relax} = 2\Gamma/\Delta_m^2 = \left\langle \Delta_B^2 \right\rangle L_0/\Delta_m^2 \qquad (2.6)$$

depends on the mean squared magnetic field parameter

$$\langle \Delta_B^2 \rangle^{1/2} = \frac{|\mu_{eff}| \left\langle \mathbf{B}^2 \right\rangle^{1/2}}{\sqrt{3}}, \qquad (2.7)$$

where μ_{eff} is defined in ref. [5] and L_0 is the domain size where the magnetic field is taken as uniform and constant. In (2.5) the quantity Δ_m

$$\Delta_m = [(V_{as} - \Delta\cos 2\theta)^2 + \Delta^2 \sin^2 2\theta]^{1/2} \qquad (2.8)$$

is the standard oscillation frequency in the supernova medium [8].

Note that (2.5) is valid when $\Gamma \gg \Delta_m$ is fulfilled and this holds in the case of a very strong r.m.s. magnetic field $\mathcal{O}$ (10^{14} - 10^{16}) Gauss.

The relaxation time in (2.6) can be much larger than the mean active neutrino collision time $t_{coll} = \Gamma_a(B \neq 0)^{-1}$. In order to see this we have used the estimate [9],

$$\Gamma_a(B \neq 0) \lesssim 2B_{14}\Gamma_a(B = 0) \qquad (2.9)$$

where B_{14} denotes the magnetic field strength in units of 10^{14} Gauss. As we can see this collision rate could be larger than $\Gamma_a(B = 0)$ by a factor $2B_{14}$. This allows us, following ref. [3], to average (2.5) over collisions so as to obtain

$$\langle P_{\nu_a \to \nu_s}(B) \rangle = \frac{\Delta^2 \sin^2 2\theta}{\langle \Delta_B^2 \rangle \, 4\Gamma_a L_0} \equiv \frac{\sin^2 2\theta_B}{2}. \qquad (2.10)$$

where we define the mixing angle in the presence of the magnetic field via

$$\sin^2 2\theta_B = \frac{\Delta^2 \sin^2 2\theta}{2\,\langle \Delta_B^2 \rangle\, \Gamma_a L_0} = \frac{x}{2}\sin^2 2\theta_m \ , \qquad (2.11)$$

in analogy with the case of zero magnetic field, where $P_{\nu_a \to \nu_s}(B \to 0) = \sin^2 2\theta_m/2$. The parameter x is defined as

$$x = \Delta_m^2/2\Gamma\Gamma_a(B \neq 0) \qquad (2.12)$$

3. Supernova Constraints

There are two ways to place constraints on neutrino oscillation parameters using astrophysical criteria, depending on the relative value of the effective sterile neutrino effective mean free path $l_s \equiv \Gamma_s^{-1} \equiv [P(\nu_a \to \nu_s)\Gamma_a]^{-1}$ and the core radius R_{core}. If the trapping condition $l_s \leq R_{core}$ is fulfilled, the ν_S are in thermodynamical equilibrium with the medium and, due to the Stefan-Boltzman law, the ratio of the sterile neutrino luminosity to that of the ordinary neutrinos,

$$\frac{Q_s}{Q_a} \simeq \left(\frac{T(R_s)}{T(R_a)}\right)^4 \left(\frac{R_s}{R_a}\right)^2 \simeq \left(\frac{\Gamma_a}{\Gamma_s}\right)^{1/2} = \left(\frac{\sin^2 2\theta_m}{2}\right)^{-1/2}, \qquad (3.1)$$

does not depend on Γ_a. In this first regime one considers surface thermal neutrino emission and sets the conservative limit $(Q_s/Q_a)_{max} \gtrsim 10$ in order to obtain the excluded region of neutrino parameters, valid for $\Delta m^2 \gtrsim$ KeV^2 [4] †

$$\sin^2 2\theta_m \lesssim 2 \times 10^{-2} \qquad (3.2)$$

† The cosmological arguments that forbid neutrino masses in the KeV range or above are not applicable in models with unstable neutrinos that decay via majoron emission [1].

In the case nonzero r.m.f. we obtain this is replaced by

$$\sin^2 2\theta_m \lesssim \frac{4\times 10^{-2}}{x} \qquad (3.3)$$

Another complementary constraint can be obtained from the requirement that in the non-trapping regime the sterile neutrino can be emitted from anywhere inside the star volume with a rate

$$\frac{dQ(B=0)}{dt} \simeq \frac{4}{3}\pi R^3_{core} n_{\nu e}\Gamma_s \langle E_s\rangle \simeq 1.4{\times}10^{55}\sin^2 2\theta_m \frac{\mathrm{J}}{\mathrm{s}}$$

which should not exceed the maximum observed integrated neutrino luminosity. For instance, for the case of SN1987A, this is $\sim 10^{46}$ J, so that one obtains the excluded region [4]

$$\sin^2 2\theta_m \gtrsim 7\times 10^{-10} \qquad (3.4)$$

In the case of a strong magnetic field $B \neq 0$ we use the known estimate for the active neutrino collision rate (2.9) and the relationship between the corresponding conversion probabilities in order to obtain the ratio of sterile neutrino volume energy losses in the presence and absence of magnetic field

$$\frac{dQ(B=0)/dt}{dQ(B\neq 0)/dt} \sim \frac{1}{xB_{14}}, \qquad (3.5)$$

where x is the small parameter in (2.12). From the last inequality we can find a region of abundances where our result for the conversion probability (2.10) is valid ($x \ll 1$) so that we obtain the excluded region

$$\sin^2 2\theta_m \gtrsim \frac{7\times 10^{-10}}{xB_{14}}\,. \qquad (3.6)$$

Note that this constraint on the neutrino parameters can be more stringent than that of (3.4). In particular, for a supernova with strong magnetic field it is possible to exclude all region of large mixing angles, if the parameter x in (2.12) is $x \leq 0.04$, as we showed in Fig. 1 of ref. [5]. This will be realized for a r.m.s. field $B_{14} \sim 10^2$ [6] and 100 MeV mean sterile neutrino energy if the abundance parameter is less than

$$\mid 3Y_e + 4Y_{\nu e} - 1 \mid \leq 0.3\times Y_e^{1/3}\rho_{14}^{-1/6}\;. \qquad (3.7)$$

This condition may indeed be realized for a stage of supernova after bounce [4,10]. Moreover, this assumption is not crucial for us, in contrast to the case of resonant neutrino spin-flip due to a neutrino magnetic moment.

4. Conclusions

The possible existence of huge random magnetic fields that might be generated during the first few seconds of neutrino emission in a supernova modifies the neutrino spectrum due to the magnetization of the medium, and thereby affect the active to sterile neutrino conversion rates. Their effect on the cooling rates may enable one to place more stringent limits than those that apply in the absence of a magnetic field. This happens despite the fact that in the presence of a large magnetic field the active to sterile neutrino conversion probability is suppressed relative to that in the zero field case due to the larger energy difference between the two diagonal entries in the neutrino evolution hamiltonian caused by the extra axial term. However, the sterile neutrino production rate could be larger in this case due to the effect of the large magnetic field. On the other hand the ratio of active and sterile neutrino thermal luminosities does not depend on the active neutrino production rate. However, the smaller the conversion probability the larger the sterile neutrino effective mean free path, and therefore they can leave the star more easily than in the case of zero magnetic field. This may lead to the exclusion of the complete large mixing angle region [5].

Acknowledgements

This paper has been supported by DGICYT under Grant number PB92-0084.

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